A numerical study on internal waves induced by a typhoon around a continental shelf

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Abstract:

Three-dimensional simulation of offshore currents induced by a traveling typhoon of strong winds and low pressure was performed assuming initial two-layer stratification of the sea water. The typhoon generates horizontal circulation involving divergence due to the Coriolis force, resulting in vertical circulation, which flows through the density interface. Accordingly, part of the interface is raised. The interface shows a wave train, which has the near-inertial period and propagates onshore at almost the same velocity as the passing velocity of the typhoon. The interfacial waves show their largest wave height just in front of the edge of the continental shelf, after which they separate into three components: transmitted, reflected, and orthogonal waves. The orthogonal waves turn left over the shelf edge in the northern hemisphere.

KEYWORDS Internal wave; typhoon; continental shelf; inertial period; 3D numerical simulation

INTRODUCTION

Hurricanes or typhoons induce strong winds and a reduction in pressure at the water surface. Aside from immediate localized effects, these disturbances also result in effects that propagate within the deeper water and towards surrounding waters, resulting not only in drifting and suction in surface layers but also in stratified flows in deeper waters. For this reason, when a typhoon passes a lake or the ocean, the water motion includes a vertical component and vertical distribution of velocity as shown by Hearn and Holloway (1990) using a numerical model, as well as stratification and mixing of density as represented in hydraulic experiments by Stevens and Imberger (1996). These characteristics cannot be evaluated when using shallow-water models usually utilized in storm-surge computations.

An open-ocean mooring observation by Brink (1989) produced a hurricane-forced vertical structure in a thermocline, where the velocity pattern displayed near-inertial phase change in western North Atlantic. These field observations which showed near-inertial motion induced by hurricanes were conducted under difficult conditions during these extreme events, such that the number of sites and period of time the data was collected over were unfortunately restricted. In this study, three-dimensional numerical calculations were performed to evaluate the water motion caused by typhoons traveling above stratified systems in offshore waters. The generation of near-inertial motion due to hurricanes was numerically simulated by Chang and Anthes (1978) etc. from a meteorological perspective, where the ocean’s motion affected the sea surface temperature changing the hurricane strength. The ocean’s motion, however, induced by hurricanes in offshore areas itself is important because it changes the offshore environment, after which generated internal waves propagate over the continental shelf and function as the boundary conditions for nearshore events modeled in the numerical simulation (e.g. Csanady, 1982). From this perspective, Keen and Allen (2000) utilized the Princeton Ocean Model to simulate hurricane-induced internal waves in areas on a shelf including the Mississippi Canyon. We extended the target domain to focus not only on the generation of near-inertial internal waves generated by a traveling typhoon in deeper offshore areas but also on their behavior around the edge of a continental shelf and propagation through shallower areas on the shelf, where we investigated seawater motion around pycnoclines as well as in the surface region.

This study aims to provide a detailed profile of large water bodies affected by extreme events such as a strong typhoon by introducing large eddy simulation.

NUMERICAL METHOD

A three-dimensional numerical model for incompressible-fluid motion is utilized to evaluate stratified flows due to both wind stress and low pressure on the sea surface. The governing equations are the equation of continuity, the Navier-Stokes equations with the Boussinesq assumption, and the advection-diffusion equation of salinity; these equations are solved using the finite difference method (Nakayama and Sato, 1999) with the LES model, the ALE method, and the CIP scheme.

The fixed boundaries including the seabed and sidewalls are non-slip boundaries. The initial water is still, without winds. Mixture and dissipation due to high surface waves or heavy rain are not considered in this paper, nor are the effects of tidal and ocean currents.

CALCULATION CONDITIONS

The origin of the vertical z-axis is on the still-water surface.

The seabed topography is as follows:

\[ h_0 = 50 \text{ m} \quad \text{where} \quad 0 \text{ km} \leq x < 100 \text{ km}, \]

\[ h_0 = 200 \text{ m} \quad \text{where} \quad 200 \text{ km} \leq x < 300 \text{ km} \quad \text{(continental shelf)}, \]

\[ h_0 = 1,000 \text{ m} \quad \text{where} \quad 300 \text{ km} \leq x \leq 900 \text{ km} \text{ or } 1,800 \text{ km}, \]

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Figure 1. Water surface elevation and horizontal velocity vectors on the water surface ($V_t = -13.5$ m/s)

Table I. Values of typhoon constants

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central pressure $p_c$</td>
<td>950 hPa</td>
</tr>
<tr>
<td>Pressure depth $\Delta p$</td>
<td>63 hPa</td>
</tr>
<tr>
<td>Radius for maximum gradient wind, $r^*$</td>
<td>70 km</td>
</tr>
<tr>
<td>Atmospheric density $\rho_{at}$</td>
<td>1.0 kg/m$^3$</td>
</tr>
<tr>
<td>Coriolis coefficient $f$</td>
<td>$8.9 \times 10^{-4}$ s$^{-1}$</td>
</tr>
<tr>
<td>Passing velocity $V_t$</td>
<td>$-13.5$ m/s or $-6.8$ m/s</td>
</tr>
<tr>
<td>Reduction coefficient $C_1$</td>
<td>0.7</td>
</tr>
<tr>
<td>Reduction coefficient $C_2$</td>
<td>1.0</td>
</tr>
<tr>
<td>Deflection angle $\alpha$</td>
<td>30°</td>
</tr>
</tbody>
</table>

where $h_0$ is the initial water depth; a uniform slope exists where $100$ km $\leq x < 200$ km. The total width of the calculation domain along the $y$-axis is 500 km.

Areas where energy is dissipated are chosen within the computational domain with a set width to make the lateral boundaries open. Inside the 30 km wide energy-dissipation areas a dissipation term, $-C_{diff}$, is assumed in the Navier-Stokes equations, where $u$ is the horizontal velocity. The constant $C_0$ should be calculated via trial and error. In the present calculations, almost the all energy transfer in the energy-dissipation areas was dissipated without reflection at the boundaries of these areas when assuming that $C_0 = 0.0234$.

Both horizontal widths of numerical grids, $\Delta x$ and $\Delta y$, are 10 km, whereas the initial vertical depth of the grids, $\Delta z_{in}$, is equal to 8 m ($-200$ m $\leq z < 0$ m) and 93 m ($z = -1000$ m). The depth changes gradually following a hyperbolic tangent relation of $z$ in other cases. The time interval $\Delta t$ is equal to 90 s.

The initial conditions were chosen by referring to observed data shown in Fig. 7.36 of Pickard and Emery (1990). We assumed initial two-layer stratification with salinities $s$ of 29.04 and 33.00 in the upper and lower layers, respectively. In calculations the density is assumed to change only with the change of salinity and the water temperature is assumed to be constant at 20 °C. The depth of the transition, where the density shows a vertically linear distribution between the upper and lower layers of the water area, is initially 12 m. An interface is defined as a plane where $s = (29.04 + 33.00)/2 = 31.02$; hereafter the upper and lower areas of this interface are called the upper and lower layers, respectively. The initial depth of upper layer, $h_0$, is 150 m.

A typhoon, whose wind and passing velocities are constant, enters, passes, and leaves the offshore area. The constant $C_0$ enters, passes, and leaves the offshore area including a low caused by the typhoon, i.e.,

$$p = p_c + \Delta p \exp(-r^2/r_0).$$

where $r$ is the distance from the typhoon center in an arbitrary horizontal plane; $p_c$ and $\Delta p$ are the central pressure and the pressure depth of typhoon, respectively. It should be noted that the gradient-wind velocity reaches a maximum value where $r = r_0$.

The gradient wind theoretically blows along constant-pressure lines when the force due to barometric gradient, centrifugal force, and Coriolis force balance as

$$\frac{\partial p}{\partial r} = \frac{\rho_{at} U_{at}}{r} + \frac{\rho_{at} f U_{at}}{r}. \quad (2)$$

where $\rho_{at}$, $U_{at}$, and $f$ are the atmospheric density, gradient-wind velocity, and Coriolis coefficient. In this paper the wind velocity is defined as the magnitude of the wind vector in a horizontal plane.

Equation (1) is substituted into Equation (2), resulting in

$$U_{at} = -\frac{rf}{2} + \sqrt{\left(\frac{rf}{2}\right)^2 + \frac{\Delta p}{\rho_{at}} \frac{r^2}{r} \exp\left(-\frac{r^2}{r_0^2}\right)}. \quad (3)$$

The wind velocity at a height of 10 m above the water surface, $U_{10}$, is assumed to be defined by Equation (4):

$$U_{10} = C_1 U_{at} + C_2 V U_{at}/U_{at}, \quad (4)$$

where $V$, and $U_{at}$ are the passing velocity of typhoon and the maximum velocity of gradient wind, respectively.

Two empirical coefficients, $C_1$ and $C_2$, are introduced (see the forecasting calculation of storm surges in Kawai et al., 2005). The second term on the right hand side of Equation (4) shows the background wind at a height of 10 m above the water surface due to the passing of the typhoon body. It should be noted that the wind vector at a height of 10 m above the water surface is inclined towards the typhoon center at an angle of $\alpha$.

Although typhoons are affected by interaction with the ocean where the strength of the typhoon is weakened by the cooling of sea surface via mechanisms.
including sea-water mixing (Wada, 2002), we assume that physical parameters of typhoons are constant. For the calculations presented in this paper, the typhoon is assumed to be large and strong such that the constants appearing in Equations (1)–(4) are given as shown in the Table I.

**CALCULATION RESULTS**

Figure 1 shows the water surface elevation and horizontal velocity vectors on the water surface when \( t = 26 \) h and 40 h, where the passing velocity of typhoon, \( V_t \), is equal to \(-13.5\) m/s. When \( t = 26 \) h, the position of typhoon center, \( x_c \), is 536.4 km, while the position of the highest water level is around \( x = 590 \) km; the center of the horizontal circulation near the water surface, where the horizontal velocity is almost zero, appears at a rearward position, where \((x, y) = (630 \text{ km}, 210 \text{ km})\). Thus the response of the ocean current appears behind a typhoon. When \( t = 40 \) h, a storm surge occurs in the nearshore area, where \( 0 \text{ km} < x < 100 \text{ km} \) and the initial water depth \( h_0 \) is 50 m; its highest water level appears where \( y \approx 400 \text{ km} \) in Figure 1b, to the right of the passing line of typhoon center, where \( x = 250 \text{ km} \).

Figure 2 shows velocity vectors and interface profiles in a vertical section, where \( y = 250 \text{ km} \) and \( t = 36 \text{ h} \) and 46 h. The timing of these snapshots was chosen to demonstrate the process most clearly. The passing velocity of typhoon is equal to \(-13.5\) m/s. In Figure 2a, the fast upward flow appears at \( x = 380 \text{ km} \), to the rear of the center of horizontal circulation in the outer area near the water surface where \( x \approx 150 \text{ km} \). The horizontal circulation is accompanied by centrifugal flows, i.e., divergence due to the Coriolis effect, resulting in vertical circulation, which runs through the density interface. Thus the interface is raised. The position of raised interface follows the typhoon traveling in a negative direction along the \( x \)-axis. For the internal-mode celerity, \( C_{iw} \), is much slower than the passing velocity of the typhoon, as described later, the raised interface does not fall readily, so that the length of the raised area extends further from the edge of continental shelf, from \( x = 300 \text{ km} \), to \( x \approx 800 \text{ km} \) where the downward flow occurs as shown in Figure 2a.

The interface shows a kind of wave train, which propagates onshore at almost the same velocity or celerity as the passing velocity of typhoon, \( V_t \). Figure 3 shows the density distribution in the vertical section where \( y = 250 \text{ km} \). In Figures 3a and 3b, where \( V_t \) is equal to \(-13.5\) m/s and \(-6.8\) m/s, the interface wavelength is about 950 km and 470 km, respectively, and the interface wave periods are about 19.5 h and 19.2 h, respectively. It should be noted that the interface wave period corresponds nearly to the inertial period: \( 2\pi / f \approx 19.6 \text{ h} \), where \( f \) is the Coriolis coefficient shown in the table, while the ‘internal-mode’ celerity \( C_{iw} \) in the present
cases is \( C_w \approx \sqrt{g(1 - \rho_i/\rho_s)h_2/(h_1 + h_2)} \approx 3.8 \text{ m/s} \) and the internal-mode periods are 69.4 h and 34.4 h in the cases of Figures 3a and 3b, respectively, where \( \rho_i \) and \( \rho_s \) are the densities of the initial waters above and below the density-transition area, respectively; \( h_i \) is the initial depth of the lower layer. The propagation of the internal-mode phenomena, including the interfacial motion and energy transfer of internal-mode waves, are comparatively so slow that the interface behavior is dominated by the flow pattern induced by the typhoon.

In a steady stratified flow, inertial oscillation can be observed due to the Coriolis force behind a topographic peak which induces upward currents. In the offshore case studies, the role of the peak as an upward-current generator is apparent when the typhoon travels at a constant velocity, such that the interface shows near-inertial oscillation due to the Coriolis force, propagating its wave profile at the traveling velocity of typhoon. Thus the Coriolis force is related to the generation of both the upward current below the typhoon and the near-inertial oscillation of the interface.

When the generated internal waves approach the continental shelf, as shown in Figure 2b, the leading wave crest of the raised interface shows its highest level, which is about 6.0 m in this vertical section, just in front of the shelf edge at \( t = 46 \text{ h} \).

Figure 4 shows the interface elevation when \( t = 52 \text{ h} \), where the passing velocity of typhoon is equal to 13.5 m/s. The leading wave crest of the raised interface separates into three components: the transmitted component, which is denoted by A in the figure, the reflected component, and the component denoted by B traveling in a negative direction along the y-axis along the shelf edge. The last component, whose energy is concentrated in a narrow area about 180 km long by 50 km wide, turns left over the shelf edge in the northern hemisphere.

According to these numerical results, we can quantify the amount of energy transmitted, reflected, and deflected, as well as periods of wave components, through modal analyses, to investigate the generation process of internal-mode waves, including internal Kelvin waves, from inertial-period waves generated in offshore areas, which requires future work.

**CONCLUSIONS**

Numerical simulation of sea water motion induced by a typhoon was performed.

The horizontal circulation of the surface waters generated divergence, i.e., centrifugal flows, due to the Coriolis force, resulting in vertical circulation through the interface. For this reason, part of the interface was raised. The raised areas of the interface displayed a wave profile, with a wave period corresponding nearly to the inertial period, following the traveling typhoon.

The interface showed its maximum level just in front of the edge of the continental shelf, after which it separates into transmitted, reflected, and orthogonal components, the last of which propagates along the edge of continental shelf.

By adapting the knowledge and outcomes of the interaction between typhoons and sea-water motion into the global-scale numerical study, it would be possible to predict precise impacts on bay areas affected by an increasing number of extreme climate events due to the global-scale climate change.

**ACKNOWLEDGMENTS**

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**SUPPLEMENTS**

1. Explanation about Movies S1, S2, and S3 is presented in Supplement.
2. Time variation of velocity vectors and an interface profile in a vertical section, where \( y = 250 \text{ km} \), is shown in Movie S1 in Supplement 2. The passing velocity of typhoon \( V_t \) is 13.5 m/s. The scaling vectors indicate \( u = 1.0 \text{ m/s} \) and \( w = 0.5 \text{ m/s} \).
3. Time variation of density distribution in a vertical section, where \( y = 250 \text{ km} \), is shown in Movie S2 in Supplement 3. The passing velocity of typhoon \( V_t \) is 13.5 m/s.
4. Same as S2 except that the passing velocity of typhoon \( V_t \) is 6.8 m/s shown in Movie S3 in Supplement 4.

**REFERENCES**


